

Title: **On developing a solvency framework for bookmakers**

Author: Dominic Cortis

Author Affiliations: Department of Mathematics, University of Leicester
Department of Insurance, FEMA, University of Malta

Contact Details: Room 104, College House, Department of Mathematics,
University of Leicester, University Road, UK LE1 7RH; dc156@le.ac.uk;
+44 (0) 116 252 1794

Acknowledgements

I thank Prof Frank Bezzina (University of Malta), Mr Ian Borg (Betsson), Mr Sandro Borg (Kyte Consultants Ltd), Mr Nick Foster FIA (University of Leicester), Mr Christian Grima (University of Malta), Dr Simon Grima (University of Malta), Prof Jeremy Levesley (University of Leicester), Mr Thomas Saliba FIA (Direct Line Group) and two anonymous referees for their valuable comments and recommendations.

Abstract: The betting industry has grown significantly but there have been no developments in creating a regulatory framework akin to the EU Solvency and Capital Requirement Directives in the Financial Services. This work derives a modular method to calculate the profit and variance of a portfolio of wagers placed with a bookmaker by subdividing these into bundles according to their likelihood size. This calls for improved risk management and regulatory set-ups similar to those of the financial services industry, which should include a minimum capital requirement for bookmakers to accept a particular number of bets – ‘A passport for taking risks’.

Key Words: Solvency, Betting, Reserves, Risk, Gambling

JEL Classification: C54, L83

1. Introduction

Actuaries are leading professionals in managing risk (BeAnActuary.org, 2015) and consequently have ventured into non-traditional research sometimes with impressive outcomes. Two such cases are Tilley's (1993) seminal paper showing the first Monte Carlo method to price a derivative option with early exercise features and Redington's development of immunisation theory (Redington, 1952).

Gambling is at times considered in a bad light, a vice not to be discussed in an academic or research setting except for evaluating its negative social implications. Yet gambling has been instrumental to the development of probability (David, 1962). Gerolamo Cardano (1501 - 1576) was an astrologer, physician, mathematician and gambler who, in an attempt to understand games of chance, first coined the classical definition of probability: given a total number n of equally possible outcomes of which m result in the occurrence of a given event, then the probability of that event occurring is $\frac{m}{n}$.

However many recognise Chevalier de Mere's (1607 - 1684) questions on gambling games, specifically his continuous losses and the problem of points, to Blaise Pascal (1623 - 1662) as the initial foundations to probability theory. Pascal famously discussed the problem of points in his correspondence with Pierre de Fermat and through their discussions these two mathematicians developed the initial stages for probability. Pascal was also instrumental in encouraging Christiaan Huygens (1629 - 1695) to write the first book on probability theory, *De Ratiociniis in Ludo Aleae* (On Reasoning in Games of Chance), in 1657 (Bernstein, 1998).

Pierre-Simon Laplace (1749 - 1827) introduced the application of probability theory to scientific and practical problems in the early nineteenth century and this spurred the developments of the theory of errors, Bayes' theory, statistics and actuarial mathematics during the same century. Subsequently Markov chains, which are extensively used in stochastic processes, were introduced in 1906 (Markov, 1906; Markov, 1971). Modern theory of probability is based on Andrey Kolmogorov's (1903 - 1987) work on measure theory. Other significant theories within probability theory during the last

century include Markowitz theory of portfolio selection (Markowitz, 1952), Monte-Carlo simulation, game theory and chaos theory.

More recently betting markets have been of great interest to economists and other researchers mainly because they can be interpreted to be a simple alternative to financial markets (Lessmann, Sung, & Johnson, 2009; Sauer, 1998; Shin, 1993; Tirole, 1982). Participants in both markets have large money at stake used to make a profit based on different beliefs and information at hand, making this a zero-sum game (Levitt, 2004). Lessman, Sung, and Johnson (2009) and Sauer (1998) extend this further by implying that gambling markets acts as simple financial markets since they provided a clearer view of pricing issues. Cases of investigations in betting markets include inquiries in insider trading (Schnytzer & Shilony, 1995; Shin, 1993; Vaughan Williams & Paton, 1997), herding behaviour (Law & Peel, 2002), a comparison of group to individual decision making (Adams & Ferreira, 2009) and market efficiency (Cortis, Hales, & Bezzina, 2013; Terrell & Farmer, 1996; Woodland & Woodland, 2001).

A further motivation is the actual size of the industry itself (Levitt, 2004). In particular the gambling industry is a key contributor to the gross domestic product (GDP) of some jurisdictions. For instance it contributes to over 10% of the Maltese GDP (Agius, 2014). On a smaller relative scale, betting on outcomes, also referred to as sports betting, is a sub-category of gambling that contributed £2.3 billion (circa 0.16%) to the United Kingdom GDP in 2011 (Deloitte, 2013).

During 2014 the sports betting industry is estimated to have made gross gambling revenue (GGR), evaluated as wagers less winning payouts, of around forty five billion euro (H2 Gambling Capital, 20 November 2014). Interactive betting, which includes internet and mobile betting, has been a key driver together with deregulation to the increase in sports betting (European Gaming and Betting Association, 2014; Fawcett, 2014; Griffiths, 2004). Indeed, there has been an increase of over 200% in GGR of interactive sports betting over the past ten years as shown in Figure 1. There are no signs of a change in the trend of a growing sports betting market as mobile and social gaming become

more popular (Torres & Goggin, 2014) and governments are viewing the regulation of the industry as possible economic and taxation gain rather than controlling a vice (Atkinson, Nichols, & Oleson, 2000). Such a development could happen in the USA where sports betting is illegal in most states by federal law. Although this has been challenged unsuccessfully by a New Jersey law that lost the case up to appeals court (Reuters, 2015), more movements are expected to focus on liberating the market in lieu of the significant amount wagered illegally in the USA.

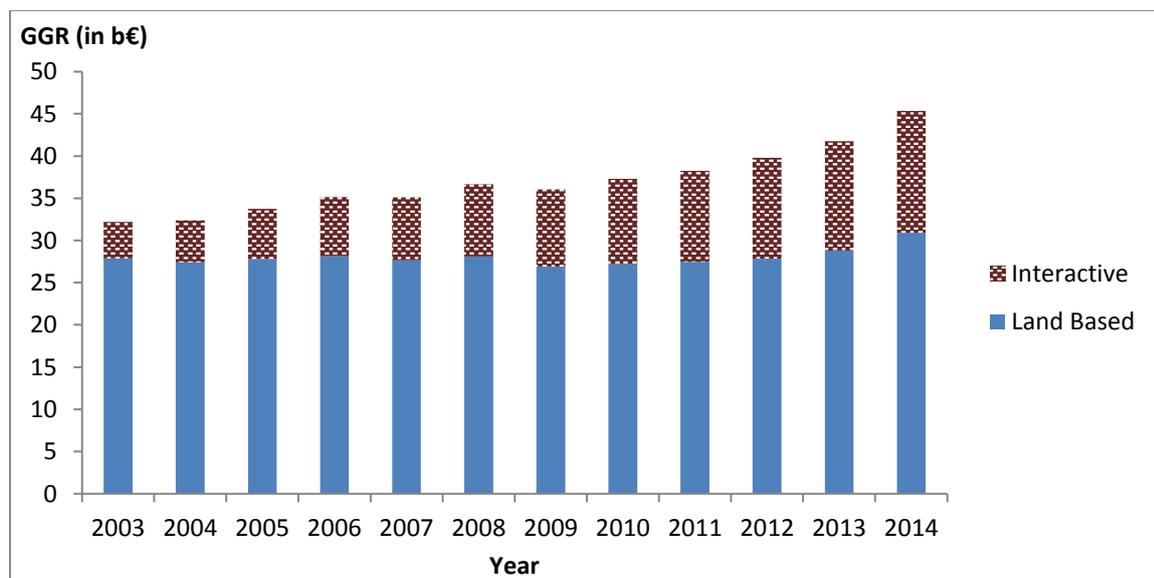


Figure 1: Estimated Yearly Gross Gambling Revenues (H2 Gambling Capital, 20 November 2014)

Due to the size of the market, the solvency of bookmakers is an issue to be addressed with a similar rigour to that of financial institutions. Bookmakers are not immune to making losses (e.g. iGaming Business, 2013, 2014). Similarly to banks, that reserve a proportion of their depositors' funds as they assume that not all depositors will withdraw their investments at the same instant, and insurers, that do not reserve the sum insured of all policies since not all extreme adverse events are likely to occur at the same time; bookmakers cannot be expected to keep a reserve equivalent to all outcomes. In this respect, this paper introduces a method to evaluate risk metrics that can be extended to limit, grant or insure a bookmaker.

Managing a bookmaker has many similarities to running an insurer. A bet can be interpreted as an insurance cover that pays out contingent on an outcome occurring. Therefore betting and insurance share the same common theme with only a few differences. Firstly there must be insurable interest for one to open up an insurance policy while one must be independent of an outcome to bet on it. A contract that pays the author \$1,000 on Queen Elizabeth II's death is a bet not an insurance policy given that the author has no insurable interest in the queen's survival. Secondly the value of a payout on an insurance claim can fluctuate, especially in a general business scenario while that of a bet tends to be fixed. For example a car accident may be a few dollars in case of a scratch to hundreds of thousands if it results in deaths. There are alternative situations, such as the fluctuating payout on a spread-bet¹ or the fixed pre-determined amount paid on an injury at work covered by a worker's compensation scheme. Finally, another difference is that the timing of a bet is usually known as it is paid at the end of an event while the trigger for an insurance payout may fluctuate. The model presented here can be considered as the equivalent of an insurer's underwriting risk under a Solvency II framework for a bookmaker.

The next section introduces the key relevant findings from other publications. These lead to the core of the paper: a method to evaluate the distribution of a portfolio of bets placed with a bookmaker and subsequently the determination of bookmaker solvency. Finally the conclusion is preceded by a discussion of challenges in applying this method as well as its possible extensions.

2. Distribution of Odds

Bookmakers set odds on outcomes of different events on which customers can place wagers on. Typically these are set in different markets since there are various outcomes from one event. For example the 1X2 market in a soccer match (the event) might result into three outcomes (home team

¹ This acts just like a forward/future.

win, draw, away team win). The same event could be used to determine the half time 1X2 market. Furthermore odds can be displayed in a variety of manners². One such method is European (or Decimal) odds whereby the odd is the reciprocal of the probability. For example an odd of two represents an implied probability of one half. Another method is American Money-Line Odds that display the amount to be wagered to win 100 units for likely events and the amount to be won on a 100 unit wager on unlikely events.

NFL						
Time	Game	Spread		Money Line	Total	
16.10	Philadelphia Eagles	-2.5	-124	-147	Over 55	Under 55
	Atlanta Falcons	+2.5	+112	+133	-103	-107
19.20	Minnesota Vikings	-2	-103	-123	Over 41.5	Under 41.5
	San Francisco 49ers	+2	-107	+111	-108	-102

Figure 2: Odds on National Football League as at 03:20CDT as shown on Pinnacle Sports

In order for bookmakers to be able to make profits, the sum of implied probabilities of all mutually exclusive outcomes for any market adds up to more than 100% (Cain, Law, & Peel, 2003; Cortis, Hales, & Bezzina, 2013; Peel & Thomas, 1992; Štrumbelj, 2014; Zafiris, 2014). The excess of this sum over 100% is called the bookmaker margin, the over-round or the vig. Typically the bookmaker margin is considered to be a constant factor that grosses up real to implied probability for all outcomes (e.g. Archontakis & Osborne, 2007; Cortis, Hales, & Bezzina, 2013; Goddard & Asimakopoulos, 2004; Zafiris, 2014) but other approaches, such as using a finite list of odds (Koch & Shing, 2008) or conversions minimizing the effects of insider trading (Shin, 1993; Štrumbelj, 2014), are also possible.

Cortis (2015) shows that for a market with n outcomes each denoted i with real probability of occurring π_i such that $\sum_{i=1}^n \pi_i = 1$ and a wager w_i made on each outcome i ; the profitability for the

² Cortis (2015) gives a detailed overview of different methods of displaying odds.

bookmaker has a mean and variance as shown in Equation 1 if a constant bookmaker margin of k is assumed,

$$P \sim N \left(\frac{k}{1+k} \sum_{i=1}^n w_i, \sum_{i=1}^n \frac{w_i^2}{(1+k)\pi_i} - \left(\frac{\sum_{i=1}^n w_i}{1+k} \right)^2 \right) \quad (1)$$

This has been further expanded such that the real probability is not required to be known by a bookmaker to have positive expected profits. For example, the odds on the match outcome for Philadelphia Eagles – Atlanta Falcons NFL match as shown in Figure 2 indicate implied probabilities of $\frac{147}{147+100} = 59.51\%$ and $\frac{100}{100+133} = 42.92\%$ for each team to win respectively. The total implied probabilities add up to 102.43%, where the excess of 2.43% is the bookmaker margin denoted as k in Equation 1. Therefore using this equation, Pinnacle Sports expects to make a profit of $\frac{2.43}{102.43} = 2.37\%$ of the total wagers made on match outcome for this case.

The findings can be extended to multiples or accumulators, which are a series of bets made on m independent events, by Equation 2. This is equivalent to Equation 1 if assuming an overall bookmaker margin of $(1 + k_m)^m - 1$. In an applied scenario this means that wagers on high odds are likely to be multiples and therefore placed at higher overall bookmaker margin. Indeed the bookmaker profits on multiples are much higher than those on normal bets³ (Cain, Law & Peel, 2003; Cortis, 2015; Zafiris, 2014).

$$P^{(m)} \sim N \left(\frac{(1 + k_m)^m - 1}{(1 + k_m)^m} \sum W, \frac{1}{(1 + k_m)^m} \sum \frac{W^2}{\pi^{(m)}} - \left(\frac{\sum W}{(1 + k_m)^m} \right)^2 \right) \quad (2)$$

where

- k_m is the geometric mean of bookmaker margin per market
- $\sum W = \sum_{i_1=1}^{n_1} \left(\sum_{i_2=1}^{n_2} \left(\dots \left(\sum_{i_m=1}^{n_m} w_{1i_1 2i_2 \dots m i_m} \right) \right) \right)$

³ An example is given in the Appendix.

- $$\sum \frac{W^2}{\pi^{(m)}} = \sum_{i_1=1}^{n_1} \left(\sum_{i_2=1}^{n_2} \left(\dots \left(\sum_{i_m=1}^{n_m} \frac{w_{i_1 i_2 \dots i_m}^2}{\pi_{i_1}^\alpha \pi_{i_2}^\alpha \dots \pi_{i_m}^\alpha} \right) \right) \right)$$
- $w_{i_1 i_2 \dots i_m}$ represents the total amount wagered on a multiple bet that pays out if all outcomes i_1, i_2 to i_m within independent events 1,2,3, ..., m respectively occur.

3. Bookmaker's Portfolio

In calculating the variance of company profits for multiples placed over the same set of m events, multiple outcomes are mutually exclusive and complete the probability space of all outcomes. However a company typically offers a significantly large number of events to place wagers on, such that little to no multiples refer to the same events. Consider the following multiple/accumulator bets made on the 2014 International Basketball Federation (FIBA) World Cup as shown in Figure 3.

- Bet α : Bet on Iran to win their match versus France @ 6.880 & Bet on Croatia to win their match versus Puerto Rico @ 1.309 for total odd 9.006
- Bet β : Bet on Puerto Rico to win versus Croatia @ 3.830 and Argentina to win versus Greece @ 3.160 for total odd 12.103
- Bet γ : Bet on Argentina to win versus Greece and Turkey to win versus Dominican Republic @ 3.16 and 1.595 respectively for total odd 5.040
- Bet δ : Bet on Lithuania to win versus Slovenia @2.030

FIBA WORLD CUP - MEN BASKETBALL - THU 9/4						
GAME			HANDICAP		MONEY LINE	TOTAL POINTS
Thu 9/4	2213	Iran	+12	1.990	6.880	Over 143.5 1.952
09:00 AM	2214	France	-12	1.917	1.137	Under 143.5 1.952
Thu 9/4	2215	Croatia	-7	1.943	1.309	Over 159.5 1.952
09:00 AM	2216	Puerto Rico	+7	1.961	3.830	Under 159.5 1.952
Thu 9/4	2217	Lithuania	+1	1.980	2.030	Over 153 1.934
12:30 PM	2218	Slovenia	-1	1.925	1.877	Under 153 1.970
Thu 9/4	2219	Turkey	-4	1.980	1.595	Over 139 1.961
12:30 PM	2220	Dominican Republic	+4	1.925	2.520	Under 139 1.943
Thu 9/4	2221	Spain	-16.5	1.917		Over 152 1.943
01:00 PM	2222	Serbia	+16.5	1.990		Under 152 1.961
Thu 9/4	2223	Argentina	+5.5	1.970	3.160	Over 154.5 1.961
01:00 PM	2224	Greece	-5.5	1.934	1.413	Under 154.5 1.943

Figure 2: Odds available on Pinnacle as at 10:49 CDT on 4th September 2014

In this case bets α and β are mutually exclusive since Croatia are playing Puerto Rico while bets β and γ have a positive correlation as they both depend on Argentina winning. Bet δ is independent of other bets. Assuming that these bets were made in the given order, bet β reduces the total variance in company profits since it acts as a hedge to bet α . On the other hand bet γ increases risk by a larger factor than bet δ .

The portfolio of bets placed with a bookmaker is constantly changing, especially with the popularity of live in-play betting (Hogg, 2013; Keogh & Rose, 2013; Robinson, 2012) which is the possibility of placing wagers of outcomes while a match is being played (Brown, 2012; Croxton & Reade, 2014). Therefore a simple estimate is required.

My proposition is to subdivide the portfolio of bets wagered with a bookmaker in bundles according to odds, evaluate the distribution of each bundle separately and then calculate the distribution of the portfolio. These bundles can be set according to the odd grids set up by the bookmaker but the odds offered to customers will increase significantly if multiples are used. Therefore a range of odds within each bundle would be preferable.

3.1. Variance per Bundle

For each bundle q , evaluate the distribution of wagers $W \sim N(\bar{W}_q, \sigma_{w_q}^2)$; the number of different wagers⁴, denoted n_q ; the geometric mean of the market spread k_q (weighted by n_q) over these bets⁵; and the arithmetic mean of odds offered π_q . The average variance per bet, denoted \overline{Var}_q , can be therefore estimated as shown in Equation 3. This is a straightforward application of Equation 1.

$$\overline{Var}_q = \bar{W}_q^2 \left(\frac{1}{(1+k_q)\pi_q} - \frac{1}{(1+k_q)^2} \right) = \frac{\bar{W}_q^2}{1+k_q} \left[\frac{1}{\pi_q} + \frac{1}{1+k_q} \right] \quad (3)$$

Since each bundle contains n bets, we can evaluate the total bundle variance Var_q (Equation 4) as the addition of all variances per bet (\overline{Var}_q). This takes into consideration the average correlation for all pairs of bets within a particular bundle, denoted r .

$$Var_q = n\overline{Var}_q + 2 \sum \left(r \sqrt{\overline{Var}_q} \sqrt{\overline{Var}_q} \right) = n\overline{Var}_q + 2 \binom{n}{2} r \overline{Var}_q \quad (4)$$

The variance of a bundle can fluctuate to anywhere between zero if all bets are perfectly negatively correlated to each other to just under $n^2\overline{Var}_q$ if bets are positively correlated to each other. In an applicative scenario, it would be recommended to back-test the correlation between all bets made within a particular bundle. The value of $\sigma_{w_1}^2$ furthermore helps in controlling the dispersion in the amount of odds as a possible consideration for bundling could be the size of the wager. Assuming an infinite number of possibilities and a finite number of bets, the example of no correlation is used, whereby $Var_q = n\overline{Var}_q$, leading to the profitability of a bundle q , denoted B_q following the distribution shown in Equation 5. One can clearly notice the similarities of the per-bundle distribution, with the distribution per event shown in Equation 1.

⁴ Bets on the same set of outcomes are to be counted as one wager. For example if two people bet different amounts on the same outcome, this is to be treated as one wager.

⁵ The rationale for using the geometric mean of the market spread is derived from the variance shown in Equation 5. The multiplicative effect of this value indicates it is safer to use the geometric mean rather than the arithmetic mean. A possible extension would be that bundles are selected such that the geometric mean, arithmetic mean and median are significantly close.

$$B_q \sim N\left(\frac{k_q}{1+k_q} n_q \bar{W}_q, n_q \bar{W}_q^2 \left(\frac{1}{(1+k)\pi_q} - \frac{1}{(1+k)^2}\right)\right) \quad (5)$$

For instance consider that a company has 300 bets of odds averaging at an odd of 1.07. The wagers have an average of 10 units and a bookmaker margin of 5%. The profits in this bundle follow the distribution $B_q \sim N(142.86, 3360.55)$ and have a probability of a loss of less than 0.7%⁶. This result is highly sensitive to the correlation coefficient. If this was considered to be 0.5, then the variance of this bundle would be 502401.36 and the probability of loss as high as 42%.⁷

3.2. Portfolio Variance

Having calculated the parameters for each bundle, one can finally calculate the joint distribution for the company's whole portfolio of bets. Assuming that b bundles are created, the total portfolio variance is $\sum_s Var_s + 2 \sum_{s>t} r_{s,t} \sqrt{Var_s Var_t}$ for $s, t \in \{1, 2, \dots, b\}$ where $r_{s,t}$ is the correlation coefficient between bundles s and t .

As an example, consider calculating the distribution of bookmaker profits for a portfolio of one hundred thousand bets uniformly subdivided over 20 bundles as shown in Table 1. It is further assumed that the bookmaker margin per bundle increases for low-likelihood outlooks in line with the longshot bias, which states that the gap between real and implied probabilities is higher for less likely outcomes (Vaughan Williams & Paton, 1997; Štrumbelj, 2014). This assumption is also consistent with the expectancies of a higher proportion of multiple/accumulator bets at higher odds. For example an odd with a likelihood of 0.01% is more likely to be a bet on a combination of outcomes from different events, such as guessing the series of winners on 10 matches, rather than one outcome from a particular event. Equation 2 shows that these bets will result in a higher betting margin per bet placed.

⁶ $1 - \Phi(2.47)$

⁷ $1 - \Phi(0.20)$

Bundle (q)	n_q	\bar{W}_q	k_q	π_q	$E(B_q)$	$\sqrt{Var_q}$
1	5000	35	5.0%	97.5%	8333	654
2	5000	35	5.5%	92.5%	9123	879
3	5000	20	6.0%	87.5%	5660	613
4	5000	15	6.5%	82.5%	4577	537
5	5000	10	7.0%	77.5%	3271	408
6	5000	10	7.5%	72.5%	3488	457
7	5000	10	8.0%	67.5%	3704	507
8	5000	10	8.5%	62.5%	3917	559
9	5000	10	9.0%	57.5%	4128	614
10	5000	10	9.5%	52.5%	4338	673
11	5000	5	10.0%	47.5%	2273	369
12	5000	5	10.5%	42.5%	2376	405
13	5000	5	11.0%	37.5%	2477	446
14	5000	5	11.5%	32.5%	2578	494
15	5000	5	12.0%	27.5%	2679	553
16	5000	5	12.5%	22.5%	2778	629
17	5000	5	13.0%	17.5%	2876	731
18	5000	2	13.5%	12.5%	1189	354
19	5000	2	14.0%	7.5%	1228	467
20	5000	2	14.5%	2.5%	1266	827

Table 1: Wagers on a Portfolio of Bets with One Bookmaker

A presumption that one can make is that closer bundles are more likely to be related than distant ones. The reasoning behind this is that, since low odds imply high likelihood while high odds imply low likelihood, these are more likely to belong to a set of mutually exclusive outcomes. Although this might not be always true. For example a market with many possible outcomes is likely to produce many high mutually exclusive odds. Notwithstanding, the correlation between different bundles can again range between -1 and 1 but as a further specimen example, it can be set as shown in Equation 6.

$$r_{s,t} \begin{cases} 0.75; & \text{if } |s - t| = 1, 2 \\ 0.50; & \text{if } |s - t| = 3, 4 \\ 0.25; & \text{if } |s - t| = 5, 6 \\ 0; & \text{if } 6 < |s - t| < 14 \\ -0.25; & \text{if } |s - t| = 14, 15 \\ -0.50; & \text{if } |s - t| = 16, 17 \\ -0.75; & \text{if } |s - t| = 18, 19 \end{cases} \quad (6)$$

The total variance is calculated as $Vector_{\sqrt{var_q}} \times Correlation Matrix \times Vector_{\sqrt{var_q}}^T$ where the correlation matrix is a 20×20 matrix showing the correlations between different bundles and

$$Vector_{\sqrt{var_q}}^1 = \begin{pmatrix} 654 \\ 879 \\ 613 \\ 537 \\ 408 \\ 457 \\ 507 \\ 559 \\ 614 \\ 673 \\ 369 \\ 405 \\ 446 \\ 494 \\ 553 \\ 629 \\ 731 \\ 354 \\ 467 \\ 827 \end{pmatrix} \quad (7)$$

The distribution of the portfolio of bets placed with the bookmaker, denoted $Port$ can therefore be evaluated as following the distribution:

$$Port \sim N(72261, 30026410) \quad (8)$$

4. Determining Solvency

The estimation of the expected profit and variance of a portfolio of bets for a bookmaker can lead to further tools for regulators and bookmakers to manage their own risk, here described as the deviation from the expected. One application could be an extension of the six sigma method (Kwak

& Anbari, 2006; Schroeder, Linderman, Liedtke, & Choo, 2008) whereby the expected profit of the portfolio needs to be at least four and a half times the standard deviation for the bookmaker to be able to operate.

Many financial regulatory regimes require a minimum of capital funds to be held by the financial institution to operate. Such regimes tend to determine this amount as a percentile measure, typically the Value-at-Risk (VaR) measure at some level of confidence. A clear example is the Solvency II insurance regime that is being introduced for the European insurance industry (Doff, 2008). The VaR evaluates the amount of capital required for the financial institution to be able to withstand extreme events. In layman terms, the VaR answers the question: What amount of capital is required for the company not to go bankrupt by $x\%$ probability?

This measure has been widely criticised from a theoretical perspective, as it seems to suggest that a small level $[(1 - x)\%]$ of failures are tolerated (Doff, 2008) and it does not have adequate mathematical properties such as sub-additivity (Acerbi & Tasche, 2002; Artzner, Delbaen, Eber, & Heath, 1999; Dhaene, et al., 2006).

Furthermore VaR does not consider the magnitude of the losses beyond its value. In this respect many recommend the use of the Expected Shortfall (ES), also referred to as the Tail-Value-at-Risk (Acerbi & Tasche, 2002; Tasche, 2002; Yamai & Yoshida, 2005). ES can be described as the expected loss in the extreme percentile beyond the VaR.

5. Challenges and Extensions

One key challenge in applying this method relates to which timeframe of bets to use: should a bookmaker apply risk measurement for all bets placed within a particular time-frame, open at a particular moment or closing at a particular time-frame? The per-bundle parameters for each bookmaker can be calculated at different times and used to project future capital requirements.

Furthermore the use of different correlation parameters may result in evaluating best-estimate, pessimistic and optimistic measures which are then utilised to rate the riskiness of each bookmaker.

On the plus side, the method presented here is modular and can be easily enriched. In the example shown here each bundle has an equivalent range of implied probability (5%) even if realistically one would not expect many outcomes with a 95-100% likelihood. Indeed the ideal number of bundles and the subdivision of these would depend on the scenario at hand. The bundling technique may provide more accurate best-estimates if subdivided in more dimensions such as the bookmaker margin, size of wagers, type of event, type of market and timeframe.

The application of any model to measure risk is bound to be imperfect (Greenspan, 2008) and therefore should not be an end-all. Following a number of model failures, many point out that risk models act more as a false sense of security rather than a proven system for insolvency prevention (Colander, et al., 2009; Danielsson, 2002). On a similar tone, many have investigated inadequacies in the models used to measure risk in the financial sector (e.g. Berkowitz & O'Brien, 2002) or discussed methods on how to assess these (Kupiec, 1995). Fundamentally financial institutions operate in a market in which their actions affect the value of market securities (Colander, et al., 2009) and financial models depend on the solvency of counter-parties since they operate in a highly leveraged environment with significant trade between the institutions.

In contrast to financial markets, the outcome of an event is not and should not be affected by a bookmaker since trade between bookmakers is currently minimal and bookmakers are not highly leveraged. The failures in application of mathematical developments in finance, such as risk models and derivatives, can also be blamed on misuse rather than simply design limitations (Bezzina & Grima, 2012; Salmon, 2012). Moreover mathematical models add insight (Geoffrion, 1976) and provide a deeper understanding of the sensitivities of output to changes (Thiele, Kurth, & Grimm, 2014). A model cannot be assumed to describe perfectly the behaviour of the underlying system but it gives us a chance to see how to react in the extreme case.

Another justification for imposed reserves on banks and insurances is that they provide services that are at time compulsory (such as car insurance) and act as a public good (Born, 2001). The consideration of the betting industry as a vice may limit developments in determining solvency regimes. There is a greater natural social desire to protect consumers who are saving their income or protecting their wealth and loved ones with banks, insurances and other financial institutions than there is to protect individuals who gamble. As an example, one would feel more empathy towards an individual whose insurance is unable to settle claims for earthquake damage to her property due to the insurer's insolvency rather than a bettor who did not receive his prize winnings due to the bookmaker's insolvency.

Notwithstanding, both deserve their compensation. Another factor is that betting markets can serve as a public good, by for example providing farmers with a likelihood of bad weather as evidence by odds on betting markets in order to hedge against the risk of low or no harvest (Hahn & Tetlock, 2006). It is also unclear whether onerous regulatory regimes would result in entry barriers that would distort the market in the current betting industry environment of mergers and take-overs (eg. Farrell, 2015).

While one can argue that any formal quantitative requirements would increase regulatory and enforcement costs, bookmakers and regulators are already significantly data intensive enterprises and should be able to cope with new quantitative improvements. Nevertheless implementation of quantitative requirements may not be as straightforward as the experience of implementing Solvency II for insurance solvency regimes in Europe has shown.

Some regulators, such as the ones based in Malta and Gibraltar, would benefit greatly from a reputational perspective by ensuring that one of their largest industries are solvent. Likewise countries where a significant number of wagers are placed, such as the United Kingdom, would also benefit by adding more rigorous measures that protect customers. Yet the greatest benefactors would most likely be bookmakers themselves. Currently the solvency is a one-by-one scenario;

usually being covered by a bank guarantee which may be an inefficient allocation of capital. Runs of this model held on different assumptions show that a bookmaker would be solvent if well diversified. That is unless the operator has some concentration of risk - say a Greek bookmaker is more at a risk of default if Greece wins the European Soccer tournament again since it would have many local sentimental bets.

The gambling industry is currently a revenue source for states and governments but the introduction of quantitative requirements may increase regulation costs. The additional cost is likely to be financed by increased fees and penalties, rather than public financing, which costs may be ultimately borne by gamblers. Lessons learned from the implementation of the insurance regime would entice any implementation of a bookmakers' solvency regime to be simpler in order to minimize costs and time to enactment.

6. Conclusion

The setting of reserves of a portfolio of bets wagered with a bookmaker is therefore a gap that needs to be addressed. Financial regulation has focused significantly on risk measurement, management and culture over the past few years (Ring, Bryce, McKinney, & Webb, 2014). Similarly as the gambling sector continues to grow; a consolidated regulatory and internal approach to risk would promote active management of own risks, increase customer protection, enhance investors' ability to analyse bookmakers and reduce regulatory arbitrage.

This paper introduced a method that measures the betting risk of a bookmaker by subdividing bets in different bundles. The model presented here is based on a number of assumptions such as distributions being mainly normal distributed, bundle sizes being equal and internal correlation for each bundle being zero. This leads to significant scope for further research which would focus on ideal bundling regime, actual experienced per-bundle correlations and challenges in setting up a risk-management framework that adapts this risk measurement technique.

References

- Acerbi, C., & Tasche, D. (2002). Expected shortfall: A natural coherent alternative to Value at Risk. *Economic Notes*, 31(2), 379-388.
- Adams, R., & Ferreira, D. (2009). Moderation in groups: Evidence from betting on ice break-ups in Alaska. *The Review of Economic Studies*, 77(3), 882-913.
- Agius, M. (2014, November 6). Remote gaming is second biggest contributor to GDP. *maltatoday*. Retrieved November 28, 2014, from http://www.maltatoday.com.mt/news/national/45886/remote_gaming_is_second_biggest_contributor_to_gdp#.VHivkfmsUm4
- Archontakis, F., & Osborne, E. (2007). Playing it safe? A Fibonacci strategy for soccer betting. *Journal of Sports Economics*, 8(3), 295-307.
- Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9(3), 203-228.
- Atkinson, G., Nichols, M., & Oleson, T. (2000). The menace of competition and gambling deregulation. *Journal of Economic Issues*, 34(3), 621-634.
- BeAnActuary.org. (2015, 09 14). *What do we do?* Retrieved from Be an Actuary: <http://www.beanactuary.org/what/do/?fa=what-do-we-do>
- Berkowitz, J., & O'Brien, J. (2002). How accurate are Value-at-Risk models at Commercial Banks? *The Journal of Finance*, 57(3), 1093-1111.
- Bernstein, P. L. (1998). *Against the Gods: The Remarkable Story of Risk*. New Jersey: Wiley, John and Sons, Incorporated.

- Bezzina, F., & Grima, S. (2012). Exploring factors affecting the proper use of derivatives: An empirical study with active users and controllers of derivatives. *Managerial Finance*, 38(4), 414-435.
- Born, P. H. (2001). Insurer profitability in different regulatory and legal environments. *Journal of Regulatory Economics*, 19(3), 211-237.
- Brown, A. (2012). Evidence of in-play insider trading on a UK betting exchange. *Applied Economics*, 44(9), 1169-1175.
- Cain, M., Law, D., & Peel, D. (2003). The favourite-longshot bias, bookmaker margins and insider trading in a variety of betting markets. *Bulletin of Economic Research*, 55(3), 263-273.
- Colander, D., Goldberg, M., Haas, A., Jeselius, K., Kirman, A., Lux, T., & Sloth, B. (2009). The financial crisis and the systemic failure of the economics profession. *Critical Review: A Journal of Politics and Society*, 21(2-3), 249-267.
- Cortis, D. (2015). Expected values and variances in bookmaker payouts: A theoretical approach towards setting limits on odds. *Journal of Prediction Markets*, 9(1).
- Cortis, D., Hales, S., & Bezzina, F. (2013). Profiting on inefficiencies in betting derivative markets: The case of UEFA Euro 2012. *The Journal of Gambling Business and Economics*, 7(1), 39-51.
- Croxton, K., & Reade, J. J. (2014). Information and Efficiency: Goal Arrival in Soccer Betting. *The Economic Journal*, 124(575), 62-91. doi:10.1111/eoj.12033
- Daniélsson, J. (2002). The emperor has no clothes: Limits to risk modelling. *Journal of Banking & Finance*, 26(7), 1273-1296.
- David, F. N. (1962). *Games, Gods and Gambling: The Origins and History of Probability and Statistical Ideas from the earliest times to the Newtonian era*. New York: Hafner Publishing Company.

- Deloitte. (2013). *The Full Picture (Second Edition): Measuring the Economic Contribution of the British Betting Industry*. London: The Association of British Bookmakers.
- Dhaene, J., Vanduffel, S., Goovaerts, M. J., Kaas, R., Tang, Q., & Vyncke, D. (2006). Risk measures and comonotonicity: A review. *Stochastic Models*, 22(4), 573-606.
- Direr, A. (2013). Are betting markets efficient? Evidence from European Football Championships. *Applied Economics*, 45(3), 343-356.
- Doff, R. (2008). A critical Analysis of the Solvency II proposals. *The Geneva Papers*, 33(2), 193-206.
- European Gaming and Betting Association. (2014). *Market Reality Factsheet*. Brussels: European Gaming and Betting Association. Retrieved October 4, 2014, from http://www.egba.eu/media/FACTSHEET_MARKET_REALITY.pdf
- Fawcett, M. (2014). Are the odds in your favour?: A film-production approach to exploring sports-betting advertising in the classroom. *Screen Education*, 73, 80-85.
- Geoffrion, A. M. (1976). The purpose of mathematical programming is insight, not numbers. *Interfaces*, 7(1), 81-92.
- Goddard, J., & Asimakopoulos, I. (2004). Forecasting football results and the efficiency of fixed-odds betting. *Journal of Forecasting*, 23(1), 51-66.
- Greenspan, A. (2008, March 16). We will never have a perfect model of risk. *The Financial Times*, p. 9. Retrieved November 19, 2014, from <http://www.ft.com/cms/s/0/eddbbcf6-f360-11dc-b6bc-0000779fd2ac.html#axzz3JVrwmIQB>
- Griffiths, M. (2004). Betting your life on it. *BMJ*, 329(7474), 1055-1056.
- H2 Gambling Capital, Global Gambling Data and Intelligence Subscription Service, Global Summary Dataset, 20 November 2014.

- Hahn, R. W., & Tetlock, P. C. (2006). A new approach for regulating information markets. *Journal of Regulatory Economics*, 29(3), 265-281.
- Hogg, R. (2013). A game of two halves ... and countless bets. In R. Roocroft, *EGR Report: Sports Betting* (p. 30). London: Pageant Gaming Media.
- iGaming Business. (2013, September 18). *Sky Bet suffers "disastrous" Champions League opening night*. Retrieved from iGaming Business: <http://www.igamingbusiness.com/news/sky-bet-suffers-disastrous-champions-league-opening-night>
- iGaming Business. (2014, January 15). *EPL weekend results cost UK bookmakers 'up to £30m'*. Retrieved from iGaming Business: <http://www.igamingbusiness.com/news/epl-weekend-results-cost-uk-bookmakers-30m>
- Keogh, F., & Rose, G. (2013, October 3). Football betting - the global gambling industry worth billions. *BBC*. Retrieved November 18, 2014, from <http://www.bbc.co.uk/sport/0/football/24354124>
- Koch, A. K., & Shing, H.-F. (2008). Bookmaker and pari-mutuel betting: Is a (reverse) favourite-longshot bias built in? *The Journal of Prediction Markets*, 2(2), 29-80.
- Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives*, 3(2), 73-84.
- Kwak, Y. H., & Anbari, F. T. (2006). Benefits, obstacles, and future of six sigma approach. *Technovation*, 26(5-6), 708-715. doi:doi:10.1016/j.technovation.2004.10.003
- Law, D., & Peel, D. A. (2002). Insider trading, herding behaviour and market plungers in the British horse-race betting market. *Economica*, 69(274), 327-338.

- Lessmann, S., Sung, M., & Johnson, J. E. (2009). Identifying winners of competitive events: A svm-based classification model for horserace prediction'. *European Journal of Operational Research*, 196(2), 569-577.
- Levitt, S. V. (2004). Why are gambling markets organised so differently than betting markets? *The Economic Journal*, 114(495), 223-246.
- Markov, A. A. (1906). Rasprostranenie zakona bol'shih chisel na velichiny, zavisyaschie drug ot druga. *Izvestiya Fiziko-matematicheskogo obschestva pri Kazanskom universitet*, 15(2), 135-156.
- Markov, A. A. (1971). Extension of the limit theorems of probability theory to a sum of variables connected in a chain. In R. Howard, *Dynamic Probabilistic Systems, volume 1: Markov Chains*. (p. Appendix B). Oxford: John Wiley & Sons.
- Markowitz, H. M. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77-91.
- Peel, D., & Thomas, D. (1992). The demand for football: Some evidence on outcome uncertainty. *Empirical Economics*, 17(2), 323-331.
- Redington, F. M. (1952). Review of the principles of life office valuations. *Journal of the Institute of Actuaries*, 78, 286-340.
- Reuters. (2015, 08 25). U.S. appeals court rules against New Jersey sports betting law. *Reuters*. Retrieved 09 13, 2015, from <http://www.reuters.com/article/2015/08/25/us-usa-new-jersey-gambling-idUSKCN0QU1WC20150825>
- Ring, P. J., Bryce, C., McKinney, R., & Webb, R. (2014). Taking notice of risk culture - The regulator's approach. *Journal of Risk Research*. doi:10.1080/13669877.2014.983944
- Robinson, P. (2012, April 28). Risky business: With the advent of online gambling, are we creating an epidemic of addiction? *Daily Mail*. Retrieved November 18, 2014, from

<http://www.dailymail.co.uk/home/moslive/article-2135649/Online-gambling-Are-creating-epidemic-addiction.html>

Salmon, F. (2012). The formula that killed Wall Street. *Significance*, 9(1), 16-20.

Sauer, R. D. (1998). The economics of wagering markets. *Journal of Economic Literature*, 36(4), 2021-2064.

Schnytzer, A., & Shilony, Y. (1995). Inside information in a betting market. *The Economic Journal*, 105(431), 963-971.

Schroeder, R. G., Linderman, K., Liedtke, C., & Choo, A. S. (2008). Six Sigma: Definition and underlying theory. *Journal of Operations Management*, 26(4), 536-554.

Shin, H. S. (1993). Measuring the incidence of insider trading in a market for state-contingent claims. *The Economic Journal*, 103(420), 1141-1153.

Štrumbelj, E. (2014). On determining probability forecasts from betting odds. *International Journal of Forecasting*, 30(4), 934-943.

Tasche, D. (2002). Expected shortfall and beyond. *Journal of Banking and Finance*, 26(7), 1519-1533.

Terrell, D., & Farmer, A. (1996). Optimal betting and efficiency in parimutuel betting markets with information costs. *The Economic Journal*, 106(437), 846-868.

Thiele, J. C., Kurth, W., & Grimm, V. (2014). Facilitating parameter estimation and sensitivity analysis of agent-based models: A cookbook using NetLogo and R. *Journal of Artificial Societies and Social Simulation*, 17(3), 11.

Tilley, J. (1993). Valuing American Options in a Path Simulation Model. *Transactions of the Society of Actuaries*, 45, 83-104.

- Tirole, J. (1982). On the possibility of speculation under rational expectations. *Econometrica*, 50(5), 1163-1181. Retrieved from <http://www.jstor.org/stable/1911868>
- Torres, C. A., & Goggin, G. (2014). Mobile social gambling: Poker's next frontier. *Mobile Media & Communication*, 2(1), 94-109.
- Vaughan Williams, L., & Paton, D. (1997). Why is there a favourite-longshot bias in British racetrack betting markets? *The Economic Journal*, 107(440), 150-158.
- Woodland, L. M., & Woodland, B. M. (2001). Market efficiency and profitable wagering in the National Hockey League: Can bettors score on longshots? *Southern Economic Journal*, 67(4), 983-995.
- Yamai, Y., & Yoshihara, T. (2005). Value-at-risk versus expected shortfall: A practical perspective. *Journal of Banking & Finance*, 29(4), 997-1015.
- Zafiris, N. (2014). When is a multiple bet better than a single? *The Journal of Gambling Business and Economics*, 8(2), 1-15.

Appendix – Multiples result in higher betting margins

Consider a scenario of three events (Event A, B, C) with two binary equally outcomes (i,ii) each. Assuming a bookmaker margin of 5%, then all odds would be at $0.5^{-1}(1.05^{-1}) = 1.905$. The wager on an accumulator/multiple bet Ai-Bi-Ci that pays out only if all three outcomes occur is $1.905^3 = 6.911$.

There are eight such possible bets (2^3). Considering the combination of such mutually accumulators/multiples as one event with eight outcomes, the total implied probability space adds up to $8(6.911^{-1}) = 1.158$ which is a betting margin of $1.05^3 - 1 = 15.8\%$.

Consequently assuming a wager of one unit on each mutually exclusive multiple outcome, the total amount wagered is eight and the payout is 6.911.

The profit in this case is $1 - \frac{6.911}{8} = 13.6\%$, equivalent to $\frac{15.8}{115.8}$.