# The Skewness of Bornhuetter-Ferguson

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## ABSTRACT

The Bornhuetter-Ferguson method is among the most popular methods used to project non-life paid or incurred triangles. For this method, T. Mack (2008) developed a stochastic model allowing the estimation of the prediction error resulting from such projections. This stochastic model involves a parametrization of the Bornhuetter-Ferguson method based on incremental triangles of incurred or paid. Hence, this parametrized method differs from the usual way in which the Bornhuetter-Ferguson is usually applied on cumulative triangles of incurred or paid. Based on this proposed stochastic model, this article provides a first approach for the estimation of the third moment, i.e. the skewness, of the resulting reserving distribution. An estimate of the third moment is useful in the context of IFRS 17 where the quantile corresponding to the addition of a risk margin on top of the best estimate will have to be disclosed. In order to apply the proposed method, a few numerical examples are provided.



KEYWORDS

Bornhuetter-Ferguson; Prediction error; Skewness; Reserving distribution; Stochastic claims reserving; IFRS 17


## 1. INTRODUCTION

After the famous Chain-Ladder method, the Bornhuetter-Ferguson method (hereinafter "BF" – see Bornhuetter/Ferguson (1972)) is one of the methods most used by practicing actuaries for the projection of non-life paid or incurred triangles. The BF method relies on two sets of parameters:

- The claims development patterns derived from the incurred or paid triangles;
- The a-priori estimates for the ultimate claims amount. These estimates can come from pricing models or from any other sources.

Using the above sets of parameters, the BF method estimates the Best Estimate of the claims liability.

Following the development of the prediction error estimate for the Chain-Ladder method (Mack (1993) and Mack (1999)), the estimate of the prediction error for the BF was proposed (see Mack(2008)). In addition, different skewness estimates for the Chain-Ladder method were developed (Salzman et al. (2012) and Dal Moro (2013)). All of these estimates are done in a distribution-free framework.

In order to complete the knowledge of the BF method in a distribution-free framework, the last missing piece is the skewness of the BF method. With the knowledge of the first three moments, it is then possible to estimate the different quantiles of the reserving distributions (see Dal Moro/Krvavych (2017)). Such quantiles will certainly be useful in the context of IFRS 17 where additional quantile disclosures will have to be published by (re)insurance companies.

This article proposes an approach to estimating this skewness relying mainly on the stochastic BF framework proposed by Mack (2008). This article will be divided into the following sections:

- A first section will describe the stochastic BF model of Mack (2008) and extend it to cover the assumptions necessary to estimate the third moment;
- A second section will provide an estimate of the skewness of the BF method per accident year;
- A third section will provide the estimate of the skewness over all accident years;
- The fourth section will provide a few numerical examples.

Remark: An excel sheet developed to estimate the presented formulae is available on the URL:
https://drive.google.com/open?id=1iRPEnD8eVOECPyR4oZI_Ezd89agVHYAa

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## 2. THE STOCHASTIC MODEL UNDERLYING THE BF METHOD

Let $C_{i,k}$ denote the cumulative claims amount (either paid or incurred) of accident year i after k years of development, $1 \leq i, k \leq n$ and $v_i$ be the premium volume of accident year i where n denotes the most recent accident year. Then $C_{i,n+1-i}$ denotes the currently known claims amount of accident year i. Let further $S_{i,k} = C_{i,k} - C_{i,k-1}$ denote the incremental claims amount (with $C_{i,0} = 0$) and $U_i$ the (unknown) ultimate claims amount of accident year i. Then $R_i = U_i - C_{i,n+1-i}$ is the (unknown true) claims reserve for accident year i. Finally, let $S_{i,n+1} = U_i - C_{i,n}$ be the incremental claims amount after development year n (tail development).

Bornhuetter/Ferguson (1972) introduced their method to estimate $R_i$ as follows:

$$\hat{R}_i^{BF} = \hat{U}_i(1 - \hat{z}_{n+1-i})$$

where $\hat{U}_i = v_i \, \hat{q}_i$ with a prior estimate $\hat{q}_i$ for the ultimate claims ratio $q_i = \frac{U_i}{v_i}$ of accident year i, $\hat{z}_k \in [0; 1]$ is the estimated percentage of the ultimate claims amount which is expected to be known after development year k.

The BF stochastic model developed in Mack (2008) relies on the following assumptions related to the increments $S_{i,k}$:

- BF1: All increments $S_{i,k}$ are independent
- BF2: There are unknown parameters $x_i$, $y_k$ such that:
  - $E(S_{i,k}) = x_i y_k$
  - $y_1 + \cdots + y_{n+1} = 1$
- BF3: There are unknown proportionality constants $s_k^2$ with $Var(S_{i,k}) = x_i s_k^2$

On the basis of these 3 assumptions, the prediction error of Bornhuetter-Ferguson can be estimated (see Mack (2008)). The prediction error, usually denoted as MSEP (Mean Squared Error of Prediction) consists of two components, the process error and the estimation error. Whereas the estimation error basically always can be calculated via the laws of error propagation, for the process error a stochastic model of the claims process was developed by T. Mack (2008).

In order to estimate the skewness of the BF method, we need a fourth assumption which is derived by analogy with the skewness estimation of the Chain-Ladder model (see Dal Moro (2013)):

- BF4: There are unknown proportionality constants $t_k^3$ with $\mathrm{K}(S_{i,k}) = x_i^{3/2} t_k^3$
  where $\mathrm{K}(S_{i,k}) = E\left[\left(S_{i,k} - E(S_{i,k})\right)^3\right]$

Following Mack (2008), we have the following with $x_1, \ldots x_n$ known:

(1)
$$\hat{y}_k = \sum_{i=1}^{n+1-k} S_{i,k} \Bigg/ \sum_{i=1}^{n+1-k} x_i$$

is a best linear unbiased estimate of $y_k$ and:

(2)
$$\hat{s}_k^2 = \frac{1}{n-k} \sum_{i=1}^{n+1-k} \frac{\left(S_{i,k} - x_i \hat{y}_k\right)^2}{x_i}$$

is an unbiased estimate of $s^2_k$.

And, following BF4, we get:

(3) $$\hat{t}_k^3 = \frac{1}{n-k}\sum_{i=1}^{n+1-k}\frac{\left(S_{i,k} - x_i\hat{y}_k\right)^3}{x_i^{3/2}}$$

is an unbiased estimate of $t^3_k$.

Having estimated the parameters $\hat{y}_k, \hat{s}_k^2$ *and* $\hat{t}_k^3$ (see Mack (2008) for details on the estimation process), we can calculate the BF claims reserve by:

$$\hat{R}_i^{BF} = \hat{U}_i(\hat{y}_{n+2-i}+\ldots+\hat{y}_{n+1}) = \hat{U}_i(1-\hat{z}_{n+1-i}) \text{ with } \hat{z}_k = \hat{y}_1+\ldots+\hat{y}_k$$

The properties of these estimators are given below:

- $\hat{y}_1,\ldots,\hat{y}_{n+1}$ are pairwise (slightly) negatively correlated as they have to add up to unity.
- $\hat{y}_1,\ldots,\hat{y}_{n+1}$ and $\hat{z}_1,\ldots,\hat{z}_{n+1}$ are independent from $\hat{U}_1,\ldots,\hat{U}_n$
- $\hat{R}_i^{BF}$ and $R_i$ are independent (due to BF1)
- $E(\hat{U}_i) = E(U_i) = x_i \text{ for } 1 \le i \le n$
- $E(\hat{y}_k) = y_k \text{ for } 1 \le k \le n+1$ and therefore $E(\hat{z}_k) = z_k \text{ for } 1 \le k \le n+1$
- $E(\hat{s}_k^2) = s_k^2 \text{ for } 1 \le k \le n+1$
- $E(\hat{t}_k^3) = t_k^3 \text{ for } 1 \le k \le n$

For the last 4 bullet points, we simply assume that the actuary's selections are unbiased.

Having described the stochastic BF model and its assumptions, the next section will be devoted to estimate the skewness of the BF method for one accident year.

## 3. SKEWNESS OF THE BF METHOD PER ACCIDENT YEAR

The mean skewness of any reserve estimate $\hat{R}_i$ is defined as:

$$SKEW(\hat{R}_i) = E\left[\left(\hat{R}_i - R_i\right)^3 \mid S_{i,1},\ldots,S_{i,n+1-i}\right]$$

Due to BF1, $\hat{R}_i^{BF}$ and $R_i$ are taken to be commonly independent from $S_{i,1},\ldots,S_{i,n+1-i}$. Hence:

$$SKEW(\hat{R}_i^{BF}) = E\left[\left(\hat{R}_i^{BF} - R_i\right)^3\right]$$

In appendix A, it is proved that:

(4) $$SKEW(\hat{R}_i^{BF}) = K(\hat{R}_i^{BF}) - K(R_i)$$

Following BF4, we have:

$$K(R_i) = K(S_{i,n+2-i})+\ldots+K(S_{i,n+1}) = x_i^{3/2}(t_{n+2-i}^3+\ldots+t_{n+1}^3)$$

which is estimated by:

$$\hat{K}(R_i) = \hat{U}_i^{3/2}(\hat{t}_{n+2-i}^3+\ldots+\hat{t}_{n+1}^3)$$

As for $K\left(\hat{R}_i^{BF}\right) = K\left[\hat{U}_i(1-\hat{z}_{n+1-i})\right]$, we use the formula below which holds when X and Y are independent:

$$K(XY) = K(X)K(Y) + K(X)E(Y)[3Var(Y) + E(Y)^2] + K(Y)E(X)[3Var(X) + E(X)^2]$$
$$+6E(X)E(Y)Var(Y)Var(X)$$

Using the above formula, we have:

$$K\left(\hat{R}_i^{BF}\right) = -\,K\left[\hat{U}_i\right]K(\hat{z}_{n+1-i}) + \,K\left[\hat{U}_i\right]E(1-\hat{z}_{n+1-i})[3Var(\hat{z}_{n+1-i}) + E(1-\hat{z}_{n+1-i})^2]$$
$$-\;K[\hat{z}_{n+1-i}]E\left(\hat{U}_i\right)\left[3Var\left(\hat{U}_i\right) + E\left(\hat{U}_i\right)^2\right] + 6\,E\left(\hat{U}_i\right)E(1-\hat{z}_{n+1-i})Var\left(\hat{U}_i\right)Var(\hat{z}_{n+1-i})$$

Estimators of $E\left(\hat{U}_i\right), Var\left(\hat{U}_i\right), E(1-\hat{z}_{n+1-i})$ *and* $Var(\hat{z}_{n+1-i})$ are provided in the article of Mack (2008) (see appendix B for details on these estimators). There remains to estimate $K\left[\hat{U}_i\right]$ *and* $K(\hat{z}_{n+1-i})$.

As for $K\left[\hat{U}_i\right]$, we have to make an assumption on the underlying distribution of $\hat{U}_i$. As a generally accepted distribution used in reserving contexts, we will therefore assume a lognormal distribution with parameters μ and σ derived from $E\left(\hat{U}_i\right)$ *and* $Var\left(\hat{U}_i\right)$. The parameters and resulting skewness are given by:

$$\sigma = \sqrt{ln\left(1 + \frac{Var(\hat{U}_i)}{E(\hat{U}_i)^2}\right)}$$

$$K\left[\hat{U}_i\right] = \left(2 + exp(\sigma^2)\right)\frac{Var(\hat{U}_i)^2}{E(\hat{U}_i)}$$

As for $K(\hat{z}_{n+1-i})$, we note that, due to the slightly negative correlations between $\hat{y}_1, \dots, \hat{y}_{n+1}$, we have to make the following approximations:

$$K(\hat{z}_k) \approx min[K(\hat{y}_1) + \ldots + K(\hat{y}_k); K(\hat{y}_{k+1}) + \ldots + K(\hat{y}_{n+1})]$$

As we have:

$$K(\hat{y}_k) = K\left(\sum_{i=1}^{n+1-k} S_{i,k} \Big/ \sum_{i=1}^{n+1-k} \hat{U}_i\right) = \sum_{i=1}^{n+1-k} K\left(S_{i,k}\right) \Big/ \left(\sum_{i=1}^{n+1-k} \hat{U}_i\right)^3$$

and:

$$\hat{K}\left(S_{i,k}\right) = \hat{U}_i^{3/2}\hat{t}_k^3$$

we finally get:

$$K(\hat{y}_k) = \hat{t}_k^3 \sum_{i=1}^{n+1-k} \hat{U}_i^{3/2} \Big/ \left(\sum_{i=1}^{n+1-k} \hat{U}_i\right)^3$$

This completes the estimation of the different elements of $SKEW\left(\hat{R}_i^{BF}\right)$.

## 4. SKEWNESS OF THE BF METHOD OVER ALL ACCIDENT YEARS

Having estimated the skewness of the BF method by accident year, we will now aggregate these elements over all accident years. In order to do so, we will use the Fleishmann polynomials (see Fleishman (1978)). First, we assume that the centralized and normalized copy of the risk value $X_i$ of the i-th class, $\hat{X}_i = \frac{X_i - E(X_i)}{E(X_i)\,CoV_{X_i}}$ (where CoV denotes the coefficient of variation), is estimated by the Fleishman polynomial structure of a standard normal random variable. In particular, we consider the following two different cases:

- $\hat{X}_i = P_2(Z_i) = a_i Z_i + b_i(Z_i^2 - 1)$ – where $Z_i$ denotes the standard normal distribution – Such a case is suitable for estimating the skewness of a risk portfolio profile when the confidence level is approximated using skewness only;
- $\hat{X}_i = P_3(Z_i) = a_i Z_i + b_i(Z_i^2 - 1) + c_i Z_i^3$ - suitable for estimating skewness and kurtosis of the risk portfolio risk profile when the confidence level is approximated using both skewness and kurtosis.

The coefficients of the polynomials $P_2$ and $P_3$ are calibrated using the method of moments by matching the second and third moments of $P_2(Z_i)$ and the second, third and fourth moments of $P_3(Z_i)$ to 1 (standard deviation of $\hat{X}_i$), $\gamma_i$ (skewness of $X_i$ ) and $\iota_i + 3$ (non-centralised or absolute kurtosis of $X_i$ ) respectively.

The coefficients of $P_2$ can be analytically expressed by solving the following system of equations:

$$\begin{cases} 1 = a_i^2 + 2b_i^2 \\ \gamma_i = 6a_i^2 b_i + 8b_i^3 \end{cases} \tag{5}$$

The system (1) is reduced to

$$\begin{cases} a_i = \sqrt{1 - 2b_i^2} \\ \gamma_i = 6b_i - 4b_i^3 \end{cases} \tag{6}$$

Such system is easily solved. The roots of the cubic equation (6) can be found using the Cardano's formula (see, e.g., Abramowitz and Stegun, 1972). If we denote:

$$\varphi = \arccos\left(-\frac{\gamma_i^3}{\sqrt{8}}\right)$$

Then, the only real root of equation (6) is:

$$b_i = \sqrt{2}\cos\left(\frac{\varphi}{3} + 4\frac{\pi}{3}\right)$$

Having estimated the above parameters of the Fleishman polynomial, we define the total reserve value across the portfolio of m risks as:

$$X_\Sigma = \sum_{i=1}^{m} X_i$$

where each i-th risk value is approximated by Fleishman polynomial of a standard normal random variable:

$$X_i \approx CE_i\left(1 + CoV_i\, P_3(Z_i)\right)$$

where CE denotes the central best estimate of X.

It is clear that: $CE_\Sigma = \sum_{i=1}^{m} CE_i$

It should also be noted that setting $c_i = 0$ reduces the problem to simply approximating:

$$X_i \approx CE_i\left(1 + CoV_i\, P_2(Z_i)\right)$$

As discussed earlier, all the standalone risks interact between each other according to a Gaussian dependence structure which linear correlations $\rho_{ij}$ (coefficients of a Gaussian copula) are given as in Mack (2008):

$$\rho_{ij} = \frac{\hat{z}_{n+1-j}(1-\hat{z}_{n+1-i})}{\hat{z}_{n+1-i}(1-\hat{z}_{n+1-j})}$$

SKEWNESS

We compute the third central moment of $X_\Sigma$:

$$E[(X_\Sigma - CE_\Sigma)^3] = E\left[\left(\sum_{i=1}^{m} \sigma_i \, P_3(Z_i)\right)^3\right]$$

$$E[(X_\Sigma - CE_\Sigma)^3] = \sum_{i=1}^{m} \sigma_i^3 \, \gamma_i + 3\sum_{ij} \sigma_i^2 \sigma_j \, E\left[P_3(Z_i)^2 P_3(Z_j)\right] + 6\sum_{ijk} \sigma_i \sigma_j \sigma_k \, E\left[P_3(Z_i) P_3(Z_j) P_3(Z_k)\right] \quad (7)$$

where $E[P_3(Z_i)^3] = \gamma_i$ as the Fleishman polynomial coefficients are calibrated so that the polynomial has Skewness $\gamma_i$ for i-th standalone risk profile. In formula (7), the summation term with multiple 3 has $\binom{m}{2}$ different sub-terms, and the summation term with multiple 6 is relevant if $m \geq 3$ and has $\binom{m}{3}$ different sub-terms.

The following components of formula (7) above are provided here only for the partial case where $P_2$ is used i.e. $c_i$=0 :

$$E\left[P_3(Z_i)^2 P_3(Z_j)\right] = 2\rho_{ij}\left(2a_i a_j b_i + \left(a_i^2 + 4b_i^2\right) b_j \rho_{ij}\right) \quad (8)$$

$$E\left[P_3(Z_i) P_3(Z_j) P_3(Z_k)\right] = 2\left(a_j a_k b_i \rho_{ij} \rho_{ik} + a_j a_i b_k \rho_{jk} \rho_{ik} + a_i a_k b_j \rho_{ij} \rho_{jk}\right) + 8 b_i b_k b_j \rho_{ij} \rho_{ik} \rho_{jk} \quad (9)$$

The Skewness $\gamma_\Sigma$ is then calculated as follows:

$$\gamma_\Sigma = \frac{E[(X_\Sigma - CE_\Sigma)^3]}{(CE_\Sigma \, CoV_\Sigma)^3} \quad (10)$$

In the numerical examples below, the parameters $a_i$, $b_i$ are provided, the correlation matrices are provided in appendix C and the values for $\sigma_i$ correspond to the msep($R_i$) which are estimated using Mack(2008) formulae. Details of calculations can be found on the excel sheet available under URL:

https://drive.google.com/open?id=1iRPEnD8eVOECPyR4oZl_Ezd89agVHYAa

## 5. NUMERICAL EXAMPLES

The formulae above were tested on three triangles (see appendix C). In all the cases, the coefficient of variation of $\widehat{U}_i$ is kept constant at 10%. The correlation matrices used for applying the Fleishmann Polynomials are also provided in appendix C.

TRIANGLE 1[2]

| Triangle 1 | | | | Fleishmann polyn. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| i | Ri BF | msep(Ri) | Skewness (Ri) | **bi** | **ai** | k | $t^3_k$ | $s^2_k$ | $y_k$ | $se(y_k)$ | $z_k$ | $se(z_k)$ |
| 2017 | 193'654'347 | 12.0% | 0.172 | 0.0288 | 0.9992 | 1 | 2'386'009 | 22'883 | 4.47% | 0.35% | 4.47% | 0.35% |
| 2016 | 172'363'095 | 12.2% | 0.141 | 0.0235 | 0.9994 | 2 | 18'165'418 | 96'444 | 9.82% | 0.77% | 14.29% | 0.85% |
| 2015 | 132'336'647 | 12.9% | 0.168 | 0.0280 | 0.9992 | 3 | -4'756'396 | 74'961 | 9.83% | 0.73% | 24.12% | 1.12% |
| 2014 | 117'146'970 | 12.9% | 0.151 | 0.0251 | 0.9994 | 4 | 5'727'410 | 166'684 | 9.21% | 1.16% | 33.33% | 1.61% |
| 2013 | 90'794'602 | 13.3% | 0.073 | 0.0121 | 0.9999 | 5 | 18'245'877 | 129'123 | 8.32% | 1.10% | 41.65% | 1.95% |
| 2012 | 96'347'577 | 12.0% | -0.355 | -0.0593 | 0.9965 | 6 | 127'640'479 | 225'383 | 6.48% | 1.57% | 48.13% | 2.23% |
| 2011 | 66'978'218 | 12.3% | 0.271 | 0.0452 | 0.9980 | 7 | -902'325 | 23'032 | 3.28% | 0.56% | 51.41% | 2.16% |
| 2010 | 52'717'174 | 11.8% | 0.208 | 0.0346 | 0.9988 | 8 | 4'199'317 | 51'181 | 3.97% | 0.93% | 55.38% | 1.95% |
| 2009 | 50'921'257 | 11.5% | 0.316 | 0.0528 | 0.9972 | 9 | -271'596 | 19'225 | 2.39% | 0.64% | 57.77% | 1.84% |
| 2008 | 37'067'585 | 11.7% | 0.303 | 0.0507 | 0.9974 | 10 | 298 | 4'798 | 2.03% | 0.37% | 59.80% | 1.80% |
| 2007 | 43'021'846 | 10.7% | 0.369 | 0.0617 | 0.9962 | 11 | -695'855 | 25'322 | 1.01% | 0.98% | 60.81% | 1.51% |
| 2006 | 27'022'245 | 10.8% | 0.365 | 0.0610 | 0.9963 | 12 | 168 | 392 | 2.71% | 0.16% | 63.52% | 1.50% |
| 2005 | 26'090'865 | 10.9% | - | 0.0000 | 1.0000 | 13 | - | - | 2.67% | 0.00% | 66.19% | 1.50% |
| Total | 1'106'462'428 | 5.53% | **0.835** | | | | | | | | | |

TRIANGLE 2

| Triangle 2 | | | | Fleishmann polyn. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| i | Ri BF | msep(Ri) | Skewness (Ri) | **bi** | **ai** | k | $t^3_k$ | $s^2_k$ | $y_k$ | $se(y_k)$ | $z_k$ | $se(z_k)$ |
| 2017 | 258'640'859 | 11.0% | 0.224 | 0.0374 | 0.9986 | 1 | 23'900 | 853 | 0.31% | 0.06% | 0.31% | 0.06% |
| 2016 | 242'359'574 | 11.1% | 0.224 | 0.0374 | 0.9986 | 2 | -786'889 | 34'579 | 5.85% | 0.42% | 6.17% | 0.42% |
| 2015 | 147'775'888 | 11.8% | 0.178 | 0.0297 | 0.9991 | 3 | 2'606'836 | 31'479 | 9.06% | 0.42% | 15.23% | 0.60% |
| 2014 | 91'588'543 | 13.1% | 0.155 | 0.0259 | 0.9993 | 4 | -1'191'271 | 20'057 | 10.04% | 0.36% | 25.27% | 0.70% |
| 2013 | 71'936'537 | 14.2% | 0.130 | 0.0217 | 0.9995 | 5 | -1'052'364 | 8'457 | 9.98% | 0.24% | 35.25% | 0.74% |
| 2012 | 122'712'262 | 12.9% | 0.154 | 0.0257 | 0.9993 | 6 | 338'178 | 17'065 | 8.20% | 0.36% | 43.44% | 0.82% |
| 2011 | 83'958'001 | 14.0% | -0.055 | -0.0091 | 0.9999 | 7 | 25'778'748 | 73'577 | 7.07% | 0.81% | 50.51% | 1.15% |
| 2010 | 72'027'374 | 13.5% | -0.242 | -0.0404 | 0.9984 | 8 | 40'375'942 | 173'275 | 5.93% | 1.35% | 56.44% | 1.77% |
| 2009 | 62'342'452 | 11.7% | -0.545 | -0.0913 | 0.9916 | 9 | 41'201'466 | 163'732 | 2.48% | 1.44% | 58.92% | 1.85% |
| 2008 | 58'602'275 | 11.7% | 0.307 | 0.0512 | 0.9974 | 10 | 613'285 | 8'138 | 1.84% | 0.36% | 60.76% | 1.81% |
| 2007 | 65'360'944 | 11.6% | 0.331 | 0.0552 | 0.9969 | 11 | -77'515 | 3'469 | 1.35% | 0.27% | 62.11% | 1.79% |
| 2006 | 65'660'028 | 10.8% | 0.365 | 0.0610 | 0.9963 | 12 | 925'613 | 30'140 | 1.90% | 0.98% | 64.01% | 1.50% |
| 2005 | 42'271'185 | 11.0% | - | 0.0000 | 1.0000 | 13 | - | - | 3.53% | 0.00% | 67.55% | 1.50% |
| Total | 1'385'235'923 | 5.25% | **0.788** | | | | | | | | | |

TRIANGLE 3

| Triangle 3 | | | | Fleishmann polyn. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| i | Ri BF | msep(Ri) | Skewness (Ri) | **bi** | **ai** | k | $t^3_k$ | $s^2_k$ | $y_k$ | $se(y_k)$ | $z_k$ | $se(z_k)$ |
| 2017 | 333'726 | 13.2% | 0.134 | 0.0224 | 0.9995 | 1 | 4 | 107 | 4.55% | 0.49% | 4.55% | 0.49% |
| 2016 | 269'832 | 13.9% | -0.042 | -0.0070 | 1.0000 | 2 | 3'703 | 282 | 12.08% | 0.82% | 16.63% | 0.96% |
| 2015 | 247'348 | 13.4% | -0.197 | -0.0328 | 0.9989 | 3 | 6'023 | 695 | 10.26% | 1.34% | 26.89% | 1.65% |
| 2014 | 219'403 | 13.6% | -0.007 | -0.0012 | 1.0000 | 4 | 2'294 | 292 | 9.41% | 0.91% | 36.30% | 1.88% |
| 2013 | 170'900 | 14.6% | 0.097 | 0.0162 | 0.9997 | 5 | 345 | 84 | 6.73% | 0.52% | 43.03% | 1.95% |
| 2012 | 164'880 | 14.7% | 0.117 | 0.0195 | 0.9996 | 6 | 244 | 118 | 4.29% | 0.64% | 47.32% | 2.06% |
| 2011 | 166'041 | 14.2% | -0.155 | -0.0259 | 0.9993 | 7 | 3'129 | 193 | 2.48% | 0.87% | 49.80% | 2.23% |
| 2010 | 163'283 | 14.3% | 0.181 | 0.0302 | 0.9991 | 8 | -18 | 50 | 2.65% | 0.47% | 52.45% | 2.28% |
| 2009 | 169'810 | 13.0% | 0.008 | 0.0013 | 1.0000 | 9 | 2'801 | 310 | 3.64% | 1.29% | 56.09% | 2.25% |
| 2008 | 175'869 | 12.7% | 0.265 | 0.0442 | 0.9980 | 10 | 104 | 68 | 1.96% | 0.67% | 58.05% | 2.14% |
| 2007 | 188'390 | 10.7% | 0.254 | 0.0423 | 0.9982 | 11 | 2'418 | 246 | 2.21% | 1.51% | 60.26% | 1.52% |
| 2006 | 123'232 | 10.8% | 0.353 | 0.0589 | 0.9965 | 12 | -1 | 3 | 1.71% | 0.23% | 61.97% | 1.50% |
| 2005 | 100'075 | 10.8% | - | 0.0000 | 1.0000 | 13 | - | - | 1.74% | 0.00% | 63.71% | 1.50% |
| Total | 2'492'791 | 5.82% | **0.27** | | | | | | | | | |

[2] Ri BF: Ultimate reserves resulting from the application of the BF method
msep(Ri): Mean Squared Error of Prediction according to Mack (2008)
$t^3_k$ : Proportionality constant between the third moment and the ultimate a priori BF reserves
$s^2_k$ : Proportionality constant between the second moment and the ultimate a priori BF reserves
$y_k$ : Incremental incurred pattern
$se(y_k)$: Standard error of $y_k$ according to Mack (2008)
$z_k$ : Cumulative incurred pattern
$se(z_k)$: Standard error of $z_k$ according to Mack (2008)

For the 3 triangles, it should be noted that a tail factor has to be added as the cumulative patterns do not reach 100% (triangle 1 tail is 33.81%, triangle 2 tail is 32.45%, triangle 3 tail is 36.29%). As for these examples, the tail factors are the simple difference between the cumulated patterns coming from the application of the Mack (2008) method and 100%. Other methods could have been chosen to review the patterns such as increasing proportionally all incremental patterns to reach 100%.

As expected, the overall resulting skewness is positive in all cases. The skewness amounts are also reasonable as the corresponding distributions would not belong to extreme distributions (e.g. Pareto) but to smoother distributions (e.g. Lognormal or Gamma). Therefore, the proposed model seems to provide robust results on these 3 triangles.

# 6. CONCLUSION

This article is a first attempt to estimate the skewness of the BF method. It relies on a few assumptions which have to be derived from external knowledge of the modelled reserving risk:

- The value of $\widehat{U}_i$;
- The coefficient of variation of $\widehat{U}_i$;
- A lognormal distribution is assumed to estimate $K[\widehat{U}_i]$;
- The slightly negative correlations between $\hat{y}_1, \dots, \hat{y}_{n+1}$ results also in some assumptions being made.

As for the three two points, the application of the proposed formulae could be refined if these parameters can be estimated externally, e.g. with the knowledge of the pricing distributions, However, such parameters are not always easily available. For the value of $\widehat{U}_i$, however, the application of the Cape-Cod method can provide a good starting point.

Overall, refining the above assumptions may provide slightly better results but we should bear in mind that, in the context of IFRS 17 and of the quarterly disclosure requirements, simple and easily implementable models should be favored. Therefore, the proposed formulae will certainly fit with IFRS 17 requirements.

Further to refinements, it should be noted that the proposed skewness estimator is sensitive to the choice of the $\widehat{U}_i$ and to the tail factor retained for the pattern estimation. The sensitivity to the choice of $\widehat{U}_i$ is the result of the formulae given in chapter 3. As for the tail factor, it comes from the application of the parametrization of the Bornhuetter-Ferguson model by T. Mack (2008) and requires the cumulative pattern to reach 100% and, as a result, to usually put a tail factor. This tail factor (which is automatically calculated in this paper) may be different from the one that an actuary would have applied based on his judgment. Such difference in the tail factor and on the overall pattern could impact the overall skewness. When applying the proposed formulae, it is therefore important to test the sensitivity of the results to both the choice of $\widehat{U}_i$ and to changes in patterns.

As a next step, we must recognize also that the BF method is usually used together with the Chain-Ladder method. Therefore, as for the Hybrid Chain-Ladder method (see Arbenz et al. (2014)), a skewness formula for the mixed BF / Chain-Ladder method should be developed in a further article.

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## APPENDIX A – Proof of equation (4)

$$SKEW(\hat{R}_i^{BF}) = E\left[\left(\hat{R}_i^{BF} - R_i\right)^3\right] = E\left(\hat{R}_i^{BF\ 3}\right) - 3E\left(\hat{R}_i^{BF\ 2} R_i\right) + 3E\left(\hat{R}_i^{BF}\ R_i^2\right) - E\left(R_i^3\right)$$

As $\hat{R}_i^{BF}$ *and* $R_i$ are independent, we have:

$$SKEW(\hat{R}_i^{BF}) = E\left(\hat{R}_i^{BF\ 3}\right) - 3E\left(\hat{R}_i^{BF\ 2}\right)E(R_i) + 3E\left(\hat{R}_i^{BF}\right)E\left(R_i^2\right) - E\left(R_i^3\right)$$

As we have defined:

$$\mathrm{K}(R_i) = E\left[\left(R_i - E(R_i)\right)^3\right] = E\left(R_i^3\right) - 3E\left(R_i^2\right)E(R_i) + 2E(R_i)^3$$

we have:

$$\begin{aligned} SKEW(\hat{R}_i^{BF}) = {} & K\left(\hat{R}_i^{BF}\right) - K(R_i) \\ & +3E\left(\hat{R}_i^{BF\ 2}\right)E\left(\hat{R}_i^{BF}\right) - 2E\left(\hat{R}_i^{BF}\right)^3 \\ & -3E\left(R_i^2\right)E(R_i) + 2E(R_i)^3 \\ & -3E\left(\hat{R}_i^{BF\ 2}\right)E(R_i) + 3E\left(\hat{R}_i^{BF}\right)E\left(R_i^2\right) \end{aligned}$$

After rearranging, we find:

$$\begin{aligned} SKEW(\hat{R}_i^{BF}) = {} & K\left(\hat{R}_i^{BF}\right) - K(R_i) \\ & + E\left(\hat{R}_i^{BF} - R_i\right)\left[3E\left(\hat{R}_i^{BF\ 2}\right) + 3E\left(R_i^2\right) - 2E\left(\hat{R}_i^{BF}\right)^2 - 2E(R_i)^2 - 2E\left(\hat{R}_i^{BF}\right)E(R_i)\right] \end{aligned}$$

As $E\left(\hat{R}_i^{BF} - R_i\right) = 0$, we finally have:

$$SKEW(\hat{R}_i^{BF}) = K\left(\hat{R}_i^{BF}\right) - K(R_i)$$

## APPENDIX B – Estimators of $E(\hat{U}_i), Var(\hat{U}_i), E(1 - \hat{z}_{n+1-i})$ *and* $Var(\hat{z}_{n+1-i})$

$E(\hat{U}_i)$ is estimated by $\nu_i\ \hat{q}_i$ where $\nu_i$ is the premium volume of accident year i and $\hat{q}_i$ *is* the ultimate claims ratio.

As mentioned in Mack (2008), $Var(\hat{U}_i)$ is best obtained from the pricing distribution (when available).

As for , $E(1 - \hat{z}_{n+1-i})$, we use: $\hat{z}_k = \hat{y}_1 + \ldots + \hat{y}_k$

Finally, $Var(\hat{z}_k) \approx min[Var(\hat{y}_1) + \ldots + Var(\hat{y}_k); Var(\hat{y}_{k+1}) + \ldots + Var(\hat{y}_{n+1})]$ where:

$$Var(\hat{y}_k) = \hat{s}_k^2 / \sum_{i=1}^{n+1-k} \hat{U}_i$$

# APPENDIX C – Triangles and data used for numerical examples

| Triangle 1 | | | | Development Year | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Accident Year | Premium | Initial LR | Ui | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 2005 | 110'940'316 | 69.6% | 77'176'365 | 4'626'711 | 7'833'956 | 9'237'575 | 4'075'137 | 10'828'853 | 2'000'000 | 1'916'455 | 1'000'000 | 2'136'161 | 873'469 | -696'077 | 1'970'249 | 2'060'435 |
| 2006 | 136'881'755 | 54.1% | 74'081'078 | 2'497'127 | 11'277'229 | 7'424'316 | 5'612'201 | 3'410'710 | 930'785 | 952'588 | 3'311'966 | 2'527'888 | 2'177'772 | 655'961 | 2'129'708 | |
| 2007 | 148'066'614 | 74.1% | 109'785'557 | 6'733'949 | 16'001'488 | 8'581'545 | 16'094'884 | 4'740'896 | 2'946'401 | 5'320'553 | 8'191'596 | 858'217 | 1'921'349 | 2'674'149 | | |
| 2008 | 142'419'083 | 64.8% | 92'216'629 | 5'495'872 | 10'634'243 | 9'357'874 | 3'687'937 | 5'459'783 | 7'244'685 | 3'514'985 | 1'278'885 | 1'130'235 | 2'201'251 | | | |
| 2009 | 141'000'201 | 85.5% | 120'589'443 | 5'513'563 | 11'892'818 | 13'879'064 | 5'892'620 | 6'906'455 | 19'110'413 | 4'127'163 | 4'465'916 | 4'691'799 | | | | |
| 2010 | 148'748'619 | 79.4% | 118'144'218 | 4'642'178 | 9'611'735 | 16'674'760 | 7'170'953 | 13'301'042 | 7'576'809 | 1'561'553 | 5'247'638 | | | | | |
| 2011 | 181'023'013 | 76.1% | 137'843'628 | 5'088'492 | 12'712'715 | 12'492'986 | 16'125'708 | 17'424'813 | 9'530'326 | 6'511'744 | | | | | | |
| 2012 | 195'545'471 | 95.0% | 185'764'666 | 5'911'892 | 14'485'648 | 15'199'655 | 20'501'488 | 15'352'506 | 10'000'000 | | | | | | | |
| 2013 | 212'536'401 | 73.2% | 155'613'255 | 7'261'343 | 13'285'662 | 16'307'615 | 12'557'638 | 11'727'223 | | | | | | | | |
| 2014 | 210'377'519 | 83.5% | 175'714'686 | 5'210'360 | 23'258'993 | 10'144'201 | 23'137'200 | | | | | | | | | |
| 2015 | 219'950'578 | 79.3% | 174'402'671 | 6'709'466 | 14'819'711 | 20'470'286 | | | | | | | | | | |
| 2016 | 230'357'704 | 87.3% | 201'091'730 | 13'556'673 | 13'519'627 | | | | | | | | | | | |
| 2017 | 238'706'732 | 84.9% | 202'706'418 | 8'255'514 | | | | | | | | | | | | |

| Triangle 2 | | | | Development Year | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Accident Year | Premium | Initial LR | Ui | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 2005 | 227'650'968 | 57.2% | 130'246'487 | 89'281 | 5'826'729 | 10'642'655 | 13'443'290 | 12'861'714 | 11'016'529 | 5'941'853 | 15'777'922 | -2'016'362 | 3'897'878 | 2'017'884 | 3'991'344 | 4'599'524 |
| 2006 | 248'800'723 | 73.3% | 182'459'114 | 253'461 | 10'141'425 | 15'502'109 | 14'653'424 | 19'038'614 | 13'526'518 | 11'920'567 | 11'637'234 | 3'152'799 | 2'832'256 | 1'596'518 | 1'957'742 | |
| 2007 | 241'098'844 | 71.6% | 172'508'136 | 285'602 | 11'280'177 | 13'467'685 | 17'570'170 | 16'533'966 | 11'438'498 | 10'882'011 | 11'879'412 | 12'716'240 | 3'081'753 | 2'959'545 | | |
| 2008 | 183'864'522 | 81.2% | 149'329'997 | 548'056 | 11'224'085 | 14'034'547 | 17'843'506 | 16'022'003 | 11'874'150 | 11'319'798 | 4'775'666 | 1'954'680 | 1'837'360 | | | |
| 2009 | 188'240'854 | 80.6% | 151'761'168 | 964'209 | 11'600'107 | 16'442'016 | 14'689'707 | 15'510'264 | 11'733'827 | 17'713'234 | 3'013'169 | 3'668'265 | | | | |
| 2010 | 196'133'095 | 84.3% | 165'366'633 | 671'876 | 10'913'835 | 16'157'793 | 14'372'512 | 17'147'487 | 14'286'587 | 11'287'031 | 9'345'582 | | | | | |
| 2011 | 194'276'106 | 87.3% | 169'661'372 | 335'616 | 8'956'636 | 12'122'076 | 17'394'437 | 17'350'796 | 16'863'903 | 10'246'783 | | | | | | |
| 2012 | 245'584'590 | 88.3% | 216'964'899 | 762'265 | 11'836'973 | 17'712'645 | 23'477'706 | 18'519'857 | 18'934'294 | | | | | | | |
| 2013 | 174'627'992 | 63.6% | 111'092'658 | 859'883 | 9'284'060 | 13'453'513 | 11'872'282 | 11'664'474 | | | | | | | | |
| 2014 | 201'255'277 | 60.9% | 122'553'304 | 1'002'904 | 8'720'759 | 13'041'075 | 12'514'625 | | | | | | | | | |
| 2015 | 273'145'637 | 63.8% | 174'317'096 | 285'729 | 7'943'122 | 15'639'699 | | | | | | | | | | |
| 2016 | 406'620'096 | 63.5% | 258'284'240 | 536'767 | 9'569'288 | | | | | | | | | | | |
| 2017 | 413'560'514 | 62.7% | 259'455'540 | 513'235 | | | | | | | | | | | | |

| Triangle 3 | | | | Development Year | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Accident Year | Premium | Initial LR | Ui | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 2005 | 306'442 | 90.0% | 275'798 | 13'487 | 50'621 | 30'204 | 33'570 | 24'290 | 12'173 | 5'826 | 10'832 | 21'869 | 10'000 | 1'048 | 4'023 | 4'801 |
| 2006 | 360'077 | 90.0% | 324'069 | 20'508 | 56'047 | 17'758 | 33'570 | 24'290 | 12'173 | 5'826 | 10'832 | 21'869 | 10'000 | 17'621 | 6'244 | |
| 2007 | 526'754 | 90.0% | 474'078 | 14'156 | 50'362 | 53'811 | 43'285 | 28'710 | 31'867 | 31'430 | 18'001 | 12'657 | 6'330 | 5'052 | | |
| 2008 | 465'852 | 90.0% | 419'267 | 30'721 | 41'275 | 19'637 | 30'725 | 28'823 | 21'106 | 4'683 | 8'735 | 5'000 | 2'938 | | | |
| 2009 | 429'724 | 90.0% | 386'751 | 7'305 | 30'942 | 29'580 | 25'095 | 17'541 | 9'402 | 8'953 | 4'637 | 7'104 | | | | |
| 2010 | 381'545 | 90.0% | 343'390 | 13'601 | 40'569 | 22'200 | 20'380 | 21'140 | 10'123 | -572 | 5'787 | | | | | |
| 2011 | 367'539 | 90.0% | 330'785 | 9'657 | 38'809 | 47'837 | 31'987 | 22'077 | 6'375 | 7'262 | | | | | | |
| 2012 | 347'770 | 90.0% | 312'993 | 7'085 | 41'073 | 55'745 | 46'024 | 17'225 | 19'820 | | | | | | | |
| 2013 | 333'314 | 90.0% | 299'982 | 18'237 | 38'891 | 51'624 | 39'890 | 29'183 | | | | | | | | |
| 2014 | 382'678 | 90.0% | 344'410 | 15'008 | 41'664 | 40'047 | 25'834 | | | | | | | | | |
| 2015 | 375'905 | 90.0% | 338'314 | 21'592 | 40'788 | 26'595 | | | | | | | | | | |
| 2016 | 359'604 | 90.0% | 323'644 | 17'061 | 33'068 | | | | | | | | | | | |
| 2017 | 388'474 | 90.0% | 349'627 | 17'291 | | | | | | | | | | | | |

| Correlation matrix for Skewness - Triangle 1 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_{ij}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 1 | 100% | 94% | 89% | 87% | 84% | 80% | 74% | 69% | 60% | 51% | 40% | 29% | 15% |
| 2 | 94% | 100% | 94% | 92% | 89% | 84% | 78% | 73% | 64% | 54% | 43% | 31% | 16% |
| 3 | 89% | 94% | 100% | 98% | 94% | 89% | 83% | 77% | 68% | 57% | 45% | 33% | 17% |
| 4 | 87% | 92% | 98% | 100% | 96% | 91% | 84% | 79% | 69% | 58% | 46% | 33% | 18% |
| 5 | 84% | 89% | 94% | 96% | 100% | 95% | 88% | 82% | 72% | 60% | 48% | 35% | 18% |
| 6 | 80% | 84% | 89% | 91% | 95% | 100% | 92% | 86% | 76% | 63% | 51% | 37% | 19% |
| 7 | 74% | 78% | 83% | 84% | 88% | 92% | 100% | 94% | 82% | 69% | 55% | 40% | 21% |
| 8 | 69% | 73% | 77% | 79% | 82% | 86% | 94% | 100% | 88% | 73% | 59% | 42% | 22% |
| 9 | 60% | 64% | 68% | 69% | 72% | 76% | 82% | 88% | 100% | 84% | 67% | 48% | 26% |
| 10 | 51% | 54% | 57% | 58% | 60% | 63% | 69% | 73% | 84% | 100% | 80% | 58% | 31% |
| 11 | 40% | 43% | 45% | 46% | 48% | 51% | 55% | 59% | 67% | 80% | 100% | 72% | 38% |
| 12 | 29% | 31% | 33% | 33% | 35% | 37% | 40% | 42% | 48% | 58% | 72% | 100% | 53% |
| 13 | 15% | 16% | 17% | 18% | 18% | 19% | 21% | 22% | 26% | 31% | 38% | 53% | 100% |

| Correlation matrix for Skewness - Triangle 2 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_{ij}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 1 | 100% | 92% | 89% | 86% | 83% | 79% | 70% | 61% | 51% | 40% | 29% | 18% | 4% |
| 2 | 92% | 100% | 96% | 93% | 90% | 85% | 76% | 66% | 55% | 44% | 32% | 19% | 4% |
| 3 | 89% | 96% | 100% | 97% | 94% | 89% | 79% | 68% | 58% | 45% | 33% | 20% | 4% |
| 4 | 86% | 93% | 97% | 100% | 96% | 91% | 81% | 70% | 59% | 47% | 34% | 21% | 5% |
| 5 | 83% | 90% | 94% | 96% | 100% | 95% | 84% | 73% | 62% | 49% | 35% | 21% | 5% |
| 6 | 79% | 85% | 89% | 91% | 95% | 100% | 89% | 77% | 65% | 51% | 37% | 23% | 5% |
| 7 | 70% | 76% | 79% | 81% | 84% | 89% | 100% | 87% | 73% | 58% | 42% | 25% | 6% |
| 8 | 61% | 66% | 68% | 70% | 73% | 77% | 87% | 100% | 84% | 66% | 48% | 29% | 6% |
| 9 | 51% | 55% | 58% | 59% | 62% | 65% | 73% | 84% | 100% | 79% | 57% | 35% | 8% |
| 10 | 40% | 44% | 45% | 47% | 49% | 51% | 58% | 66% | 79% | 100% | 73% | 44% | 10% |
| 11 | 29% | 32% | 33% | 34% | 35% | 37% | 42% | 48% | 57% | 73% | 100% | 60% | 13% |
| 12 | 18% | 19% | 20% | 21% | 21% | 23% | 25% | 29% | 35% | 44% | 60% | 100% | 22% |
| 13 | 4% | 4% | 4% | 5% | 5% | 5% | 6% | 6% | 8% | 10% | 13% | 22% | 100% |

| Correlation matrix for Skewness - Triangle 3 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_{ij}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 1 | 100% | 96% | 93% | 89% | 85% | 79% | 75% | 72% | 66% | 57% | 46% | 34% | 16% |
| 2 | 96% | 100% | 96% | 92% | 89% | 82% | 78% | 74% | 68% | 59% | 48% | 35% | 17% |
| 3 | 93% | 96% | 100% | 96% | 92% | 85% | 81% | 77% | 71% | 61% | 49% | 36% | 18% |
| 4 | 89% | 92% | 96% | 100% | 96% | 89% | 85% | 81% | 74% | 64% | 52% | 38% | 19% |
| 5 | 85% | 89% | 92% | 96% | 100% | 93% | 88% | 84% | 77% | 67% | 54% | 40% | 19% |
| 6 | 79% | 82% | 85% | 89% | 93% | 100% | 95% | 90% | 83% | 72% | 58% | 43% | 21% |
| 7 | 75% | 78% | 81% | 85% | 88% | 95% | 100% | 95% | 87% | 76% | 61% | 45% | 22% |
| 8 | 72% | 74% | 77% | 81% | 84% | 90% | 95% | 100% | 92% | 80% | 64% | 47% | 23% |
| 9 | 66% | 68% | 71% | 74% | 77% | 83% | 87% | 92% | 100% | 87% | 70% | 51% | 25% |
| 10 | 57% | 59% | 61% | 64% | 67% | 72% | 76% | 80% | 87% | 100% | 80% | 59% | 29% |
| 11 | 46% | 48% | 49% | 52% | 54% | 58% | 61% | 64% | 70% | 80% | 100% | 74% | 36% |
| 12 | 34% | 35% | 36% | 38% | 40% | 43% | 45% | 47% | 51% | 59% | 74% | 100% | 49% |
| 13 | 16% | 17% | 18% | 19% | 19% | 21% | 22% | 23% | 25% | 29% | 36% | 49% | 100% |